UNIVERSIDADE FEDERAL DO RIO DE JANEIRO
Centro de Ciências Matemática e da Natureza
Instituto de Matemática

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CONCURSO PARA PROFESSOR ADJUNTO A – IM
EDITAL 54/2024

QUESTÃO 2)

(1) Seja $S \subset \mathbb{R}^3$ uma superfície regular, orientada com o campo de vetores normais unitários $\vec{n}$, parametrizada por uma aplicação $\varphi : D \to \mathbb{R}^3$, $D \subset \mathbb{R}^2$, de classe C^2 $\left(\frac{\partial \varphi}{\partial u} \times \frac{\partial \varphi}{\partial v} \neq 0 \text{ em } D \right)$, tal que a fronteira ∂S é uma curva de classe C^1 por partes orientada positivamente.

Seja $\vec{F} = (F_1, F_2, F_3) : U \to \mathbb{R}^3$ um campo vetorial de classe C^1 tal que $S \subset U$, onde U é aberto de $\mathbb{R}^3$.

Então

$$\iint_S (\text{rot}\, \vec{F} \cdot \vec{n})\, dS = \oint_{\partial S} \vec{F} \cdot d\vec{r} \quad \ldots (Rot)$$

(rotacional)

(1') Seja $S \subset \mathbb{R}^3$ uma superfície regular fechada, orientada com o campo de vetores normais unitários $\vec{n}$, que envolve uma região sólida $R \subset \mathbb{R}^3$ tal que $\partial R = S$.

Seja $\vec{F} = (F_1, F_2, F_3) : U \subset \mathbb{R}^3 \to \mathbb{R}^3$ um campo vetorial de classe C^1, onde U é um aberto de $\mathbb{R}^3$ que contém E e ∂S.

Então,

$$\iiint_E (\text{div}\, \vec{F})\, dV = \iint_{\partial S} \vec{F} \cdot d\vec{n} \quad (\text{DIVERGÊNCIA}). \quad \ldots (\text{DIV})$$

FAVOR **NÃO** ESCREVER NO VERSO!
NÃO ULTRAPASSAR A LINHA DA MARGEM!

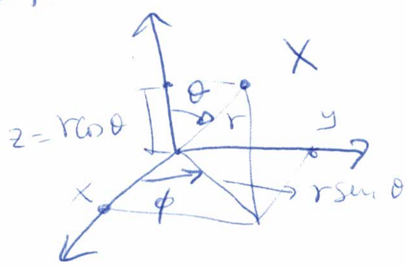
Questão 2)

(2) Observe que, se $\vec{F} = \nabla u = \left(\frac{\partial}{\partial x}u, \frac{\partial}{\partial y}u, \frac{\partial}{\partial z}u\right)$, e daí

$$\operatorname{div}\vec{F} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right) = \Delta u.$$

▷ Do T. da Divergência (DIV), temos

$$\iiint_R \Delta u \, dx\,dy\,dz = \iiint_R \operatorname{div}\vec{F}\, dx\,dy\,dz = \iint_{\partial R} \vec{F}\cdot d\vec{n} = \iint_{\partial R} \nabla u \cdot d\vec{n}.$$

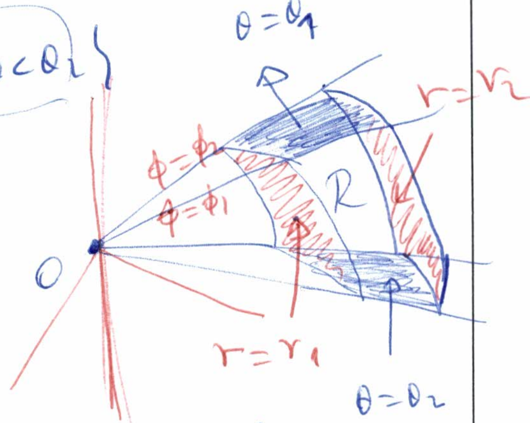


(3) Observe que: (i) $x^2+y^2+z^2 = r^2$,

segundo a notação (ii) $\cos\theta = \frac{z}{r}$

(iii) $\tan\phi = \frac{y}{x}$

▷ $R = \{(r,\phi,\theta) \,/\, r_1 < r < r_2,\ \phi_1 < \phi < \phi_2,\ \theta_1 < \theta < \theta_2\}$



▷ FASES: $F_{\theta_1} = \partial R \cap \{(r,\phi,\theta)\,;\ \theta = \theta_1\}$

$F_{\theta_2} = \partial R \cap \{(r,\phi,\theta)\,;\ \theta = \theta_2\}$

▷ Contribuição de F_{θ_1}:

$$\iint_{F_{\theta_1}} \nabla u \cdot d\vec{n} = \int_{r_1}^{r_2}\int_{\phi_1}^{\phi_2} \nabla u(\varphi^{\theta_1}(r,\phi)) \cdot (\varphi^{\theta_1}_r \times \varphi^{\theta_1}_\phi)\, d\phi\, dr, \ \text{(A1)}$$

onde $\varphi^{\theta_1}(r,\phi) = \varphi(r,\phi,\theta_1)$, $\varphi(r,\phi,\theta) = (x(r,\phi,\theta), y(r,\phi,\theta), z(r,\phi,\theta))$

Observe que: Se $u = u(r,\phi,\theta) = u(r(x,y,z), \phi(x,y,z), \theta(x,y,z))$

então
(✱) $\dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial r}\cdot\dfrac{\partial r}{\partial x} + \dfrac{\partial u}{\partial \phi}\cdot\boxed{\dfrac{\partial \phi}{\partial x}} + \dfrac{\partial u}{\partial \theta}\cdot\dfrac{\partial \theta}{\partial x}$,

e de (i): $2x = 2r\cdot\dfrac{\partial r}{\partial x} \Rightarrow \boxed{\dfrac{\partial r}{\partial x} = \dfrac{x}{r}}$

▷ De (iii) $\sec^2\phi\cdot\dfrac{\partial \phi}{\partial x} = -\dfrac{y}{x^2} \Rightarrow \dfrac{\partial \phi}{\partial x} = \dfrac{-y}{x^2}\cdot\cos^2\phi = \dfrac{-y}{x^2}\cdot\dfrac{x^2}{x^2+y^2} = \dfrac{-y}{x^2+y^2}$.

$\Rightarrow \dfrac{\partial \phi}{\partial x} = \dfrac{-y}{x^2+y^2}$.

▷ De (ii), $-\operatorname{sen}\theta\cdot\dfrac{\partial \theta}{\partial x} = z\cdot\dfrac{\partial}{\partial x}\left(\dfrac{1}{r}\right) = z\cdot\dfrac{-\frac{\partial}{\partial x}r}{r^2} = -\dfrac{z}{r^2}\cdot\dfrac{x}{r} = -\dfrac{xz}{r^3}$

$\Rightarrow \boxed{\dfrac{\partial \theta}{\partial x} = \dfrac{xz}{r^3}\cdot\dfrac{1}{\operatorname{sen}\theta} = \dfrac{xz}{r^3}\cdot\dfrac{r}{\sqrt{r^2-z^2}} = \dfrac{xz}{r^2\sqrt{r^2-z^2}}}$

(✱) $\dfrac{\partial u}{\partial y} = \dfrac{\partial u}{\partial r}\cdot\boxed{\dfrac{\partial r}{\partial y}} + \dfrac{\partial u}{\partial \phi}\cdot\boxed{\dfrac{\partial \phi}{\partial y}} + \dfrac{\partial u}{\partial \theta}\cdot\boxed{\dfrac{\partial \theta}{\partial y}}$.

▷ Observe: De (i): $2y = 2r\dfrac{\partial r}{\partial y} \Rightarrow \boxed{\dfrac{\partial r}{\partial y} = \dfrac{y}{r}}$.

▷ De (iii) $\sec^2\phi\cdot\dfrac{\partial \phi}{\partial y} = \dfrac{1}{x} \Rightarrow \dfrac{\partial \phi}{\partial y} = \dfrac{1}{x}\cdot\cos^2\phi = \dfrac{1}{x}\cdot\dfrac{x^2}{x^2+y^2} = \dfrac{x}{x^2+y^2}$

▷ De (ii): $-\operatorname{sen}\theta\cdot\dfrac{\partial \theta}{\partial y} = z\cdot\dfrac{\partial}{\partial y}\left(\dfrac{1}{r}\right) = z\cdot\dfrac{-\frac{\partial r}{\partial y}}{r^2} = \dfrac{-z}{r^2}\cdot\dfrac{y}{r} = \dfrac{-zy}{r^3}$.

$\longrightarrow \dfrac{\partial \theta}{\partial y} = +\dfrac{zy}{r^3}\cdot\dfrac{1}{\operatorname{sen}\theta} = \dfrac{zy}{r^3}\cdot\dfrac{r}{\sqrt{r^2-z^2}}$.

(✗) $\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial z} + \frac{\partial u}{\partial \phi} \cdot \frac{\partial \phi}{\partial z} + \frac{\partial u}{\partial \theta} \frac{\partial \theta}{\partial z}$.

▷ De (i), $2z = 2r \cdot \frac{\partial r}{\partial z} \to \boxed{\frac{\partial r}{\partial z} = \frac{z}{r}}$

▷ De (iii) $\frac{\partial \phi}{\partial z} = 0$

▷ De (ii) $-\sin\theta \cdot \frac{\partial \theta}{\partial z} = \frac{1}{r} \Rightarrow \frac{\partial \theta}{\partial z} = -\frac{1}{r} \cdot \frac{1}{\sin\theta} = -\frac{1}{r} \cdot \frac{r}{\sqrt{r^2 - z^2}}$

$\Rightarrow \frac{\partial \theta}{\partial z} = \frac{-1}{\sqrt{r^2 - z^2}}$. ⟶ Substituir estas derivadas em (✗) $\underline{(A_1)}$

▷ Similar para F_{θ_2}:

▷ FASES: $F_{r_1} = \partial F \cap \{(r, \phi, \theta); r = r_1\}$
$F_{r_2} = \partial F \cap \{(r, \phi, \theta), r = r_2\}$

▷ $\iint_{F_{r_1}} \nabla u \cdot d\vec{n} = \int_{\theta_1}^{\theta_2} \int_{\phi_1}^{\phi_2} \nabla u(\varphi^{r_1}(\phi, \theta)) \cdot \varphi^{r_1}_{\phi} \times \varphi^{r_1}_{\theta} \, d\phi \, d\theta$, -- (A_2)

onde $\varphi^{r_1}(\phi, \theta) = \varphi(r_1, \phi, \theta)$. Substituir as derivadas $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$ e $\frac{\partial u}{\partial z}$ calculadas antes. Similar para F_{r_2}.

▷ FASES: $F_{\phi_1} = \partial F \cap \{(r, \phi, \theta); \phi = \phi_1\}$
$F_{\phi_2} = \partial F \cap \{(r, \phi, \theta); \phi = \phi_2\}$

$$\iint_{F_{\phi_1}} \nabla u \cdot d\vec{n} = \int_{r_1}^{r_2}\int_{\theta_1}^{\theta_2} \nabla u(\varphi^{\phi_1}(r,\theta)) \cdot \partial_r \varphi^{\phi_1} \times \partial_\theta \varphi^{\phi_1}\, dr\, d\theta \quad \ldots (A3)$$

▷ Substituir as derivadas $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$, e $\frac{\partial u}{\partial z}$ anteriores.

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(5) $$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial u}{\partial \phi}\cdot\frac{\partial \phi}{\partial x} + \frac{\partial u}{\partial \theta}\cdot\frac{\partial \theta}{\partial x}.$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial r}\right)\cdot\frac{\partial r}{\partial x} + \frac{\partial u}{\partial r}\cdot\frac{\partial^2 r}{\partial x^2} + \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial \phi}\right)\cdot\frac{\partial \phi}{\partial x} + \frac{\partial u}{\partial \phi}\cdot\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial \theta}\right)\cdot\frac{\partial \theta}{\partial x} + \frac{\partial u}{\partial \theta}\cdot\frac{\partial^2 \theta}{\partial x^2}$$

$$= \left[\frac{\partial^2 u}{\partial r^2}\cdot\frac{\partial r}{\partial x} + \frac{\partial^2 u}{\partial \phi\partial r}\cdot\frac{\partial \phi}{\partial x} + \frac{\partial^2 u}{\partial \theta\partial r}\cdot\frac{\partial \theta}{\partial x}\right]\cdot\frac{\partial r}{\partial x} + \frac{\partial u}{\partial r}\cdot\frac{\partial^2 r}{\partial x^2} +$$
$$\left[\frac{\partial^2 u}{\partial r\partial \phi}\frac{\partial r}{\partial x} + \frac{\partial^2 u}{\partial^2 \phi}\cdot\frac{\partial \phi}{\partial x} + \frac{\partial^2 u}{\partial \theta\partial \phi}\cdot\frac{\partial \theta}{\partial x}\right]\cdot\frac{\partial \phi}{\partial x} + \frac{\partial u}{\partial \phi}\cdot\frac{\partial^2 \phi}{\partial x^2} +$$
$$\left[\frac{\partial^2 u}{\partial r\partial \theta}\cdot\frac{\partial r}{\partial x} + \frac{\partial^2 u}{\partial \phi\partial \theta}\cdot\frac{\partial \phi}{\partial x} + \frac{\partial^2 u}{\partial \theta^2}\cdot\frac{\partial \theta}{\partial x}\right]\cdot\frac{\partial \theta}{\partial x} + \frac{\partial u}{\partial \theta}\cdot\frac{\partial^2 \theta}{\partial x^2}$$

▷ Similar cálculo para $\frac{\partial u}{\partial y}$:

$$\frac{\partial^2 u}{\partial y^2} = \left[\frac{\partial^2 u}{\partial r^2}\frac{\partial r}{\partial y} + \frac{\partial^2 u}{\partial \phi\partial r}\cdot\frac{\partial \phi}{\partial y} + \frac{\partial^2 u}{\partial \theta\partial r}\cdot\frac{\partial \theta}{\partial y}\right]\cdot\frac{\partial r}{\partial y} + \frac{\partial u}{\partial r}\cdot\frac{\partial^2 r}{\partial y^2} +$$
$$\left[\frac{\partial^2 u}{\partial r\partial \phi}\cdot\frac{\partial r}{\partial y} + \frac{\partial^2 u}{\partial^2 \phi}\cdot\frac{\partial \phi}{\partial y} + \frac{\partial^2 u}{\partial \theta\partial \phi}\cdot\frac{\partial \theta}{\partial y}\right]\cdot\frac{\partial \phi}{\partial y} + \frac{\partial u}{\partial \phi}\cdot\frac{\partial^2 \phi}{\partial y^2} +$$
$$\left[\frac{\partial^2 u}{\partial r\partial \theta}\cdot\frac{\partial r}{\partial y} + \frac{\partial^2 u}{\partial \phi\partial \theta}\cdot\frac{\partial \phi}{\partial y} + \frac{\partial^2 u}{\partial \theta^2}\cdot\frac{\partial \theta}{\partial y}\right]\frac{\partial \theta}{\partial y} + \frac{\partial u}{\partial \theta}\cdot\frac{\partial^2 \theta}{\partial y^2}$$

▷ Similar para $\frac{\partial^2 u}{\partial z^2}$

$\Rightarrow \Delta u$ = expressão em função de $\frac{\partial u}{\partial r}, \frac{\partial u}{\partial \theta}, \frac{\partial u}{\partial \phi}$.

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QUESTÃO 3

(1) ~~Seja $(\Omega, \mathcal{A}, \mathbb{P})$ um espaço de Probabilidade.~~

Seja $(X_i)_i$ uma sequência de variáveis aleatórias em L^2, independentes e uniformemente distribuídas, com média μ e variança σ^2. Então,

(i) A distribuição da variável aleatória

$$\frac{1}{\sigma\sqrt{n}}\sum_{i=1}^{n}(X_i-\mu)$$ converge fraca* à distribuição da variável aleatória Gaussiana (NORMAL) com média μ e variança σ, ν_μ^σ.

(ii) $$\lim_{n\to\infty}\mathbb{P}\left[\frac{1}{\sigma\sqrt{n}}\sum_{i=1}^{n}(X_i-\mu)\le a\right] = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{a} e^{t} = \mathbb{P}\left[\nu_\mu^\sigma \le a\right]$$

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(2) Esboço a Prova:

Este teorema é baseado em alguns Lemas.

LEMA 1. Seja $\mu_k, \mu \in \mathcal{M}(\mathbb{R}^n)$ tal que $\|\hat{u}_k\| \le c, \forall k$ e $\hat{u}_k \to \hat{u}$. Então μ_k converge fraca* a μ.

Observe $\mathcal{M}(\mathbb{R}^n) = C_0(\mathbb{R}^n)^*$, $C_0(\mathbb{R}^n)$ = funções contínuas que anulam-se no infinito.

▷ A prova disto é baseada na densidade de S em $C_0(\mathbb{R}^n)$ e na fórmula de inversão da transformada de Fourier.

▷ De fato, se $f \in S$,

$$\int f \, d\mu_k = \int (f^\vee)^\wedge d\mu_k = \int \check{f} \hat{\mu}_k \longrightarrow \int \check{f} \hat{\mu} = \int f \, d\mu,$$

onde esta convergencia vale pelo Teorema da convergencia dominada, pois $\check{f} \in L^1$ e $\|\hat{\mu}_k\| \leq c$, $\forall k$.

▷ Como S é denso em $C_0(\mathbb{R}^n)$, segue que

$$\int f \, d\mu_k \to \int f \, d\mu, \quad \forall f \in C_0(\mathbb{R}^n).$$

LEMA 2: Seja λ uma medida de probabilidade de Borel tal que $\int t \, d\lambda(t) = 0$ e $\int t^2 d\lambda(t) = 1$. Denote $(\lambda^*)^n = \lambda * \dots * \lambda$ (n-vezes) e defina $\lambda_n(E) = (\lambda^{*})^n(\sqrt{n} E)$. Então, λ_n converge fraco* a ν_0^1.

Prova: Pelo Lema 1, basta provar que $\hat{\lambda}_n \longrightarrow \hat{\nu}_0^1$:

De fato: Primeiro, $\hat{\lambda}(\xi) = \int e^{-2\pi i \xi \cdot x} d\lambda(x)$. Pelo Teorema de Convergencia Dominada, ~~~~

$\hat{\lambda}(0) = 1$, $\hat{\lambda}'(0) = 0$, $\hat{\lambda}''(0) = (-2\pi i)^2 = -4\pi^2$.

▷ Pela Fórmula de Taylor:

$$\hat{\lambda}(\xi) = \hat{\lambda}(0) + \hat{\lambda}'(0)\xi + \hat{\lambda}''(0)\frac{\xi^2}{2} + o(\xi^2),$$

$$= 1 - 2\pi^2\xi^2 + o(\xi^2), \text{ onde } \lim_{\alpha\to 0}\frac{o(\alpha)}{\alpha} = 0$$

▷ Observe também, que:

$$\hat{\lambda}_n(\xi) = \left[\hat{\lambda}\left(\frac{\xi}{n^{\frac{1}{2}}}\right)\right]^n$$

$$\hat{\lambda}_n(\xi) = \left[1 - 2\pi^2 \cdot \frac{\xi^2}{n} + o\left(\frac{\xi^2}{n}\right)\right]^n$$

▷ Daí, como $\log(1+z) = z + o(z)$, segue que

$$\log \hat{\lambda}_n(\xi) = n\left[\log\left(1 - 2\pi^2\frac{\xi^2}{n} + o\left(\frac{\xi^2}{n}\right)\right)\right]$$

$$= n\left[-2\pi^2\frac{\xi^2}{n} + o\left(\frac{\xi^2}{n}\right) + o\left(-2\pi^2\frac{\xi^2}{n} + o\left(\frac{\xi^2}{n}\right)\right)\right]$$

$$\log \hat{\lambda}_n(\xi) = -2\pi^2\xi^2 + n\left[o\left(\frac{\xi^2}{n}\right) + o\left(-2\pi^2\frac{\xi^2}{n} + o\left(\frac{\xi^2}{n}\right)\right)\right]$$

▷ Fazendo $n\to\infty$, temos que

$$\lim_{n\to\infty}\log\hat{\lambda}_n(\xi) = -2\pi^2\xi^2 \Rightarrow \hat{\lambda}_n(\xi) \to e^{-2\pi^2\xi^2}.$$

∴ Pelo Lema 1, temos que $\lambda_n \xrightarrow{\text{fraco}^*} \nu_0^1$.

Prova do T. Central do Limite:

▷ Basta observar que, se λ é a distribuição comum das variáveis aleatórias X_i, então λ tem media 0 e variança 1, daí,

$$\int t\, d\lambda(t) = 0 \quad e \quad \int t^2 d\lambda(t) = 1$$

▷ Isto é λ satisfaz as hipóteses do Lema 2.

Portanto, λ_n, definido por $\lambda_n(E) = (\lambda^*)^n(\sqrt{n}E)$, converge fraco* a ν_0^1.

$X_1 + \dots + X_n \in \sqrt{n}E$

▷ Por outro lado,

$$\lambda_n(E) = \int \cdots \int \chi_{\sqrt{n}E}(X_1 + \dots + X_n)\, d\lambda(X_1) \dots d\lambda(X_n)$$

$$= \int \cdots \int \chi_E\left(\frac{X_1 + \dots + X_n}{\sqrt{n}}\right) d\lambda(X_1) \dots d\lambda(X_n)$$

▷ Existe outro lema que garante que, se $\lambda_n \xrightarrow{\text{fraco}^*} \nu_1^0$, então as suas distribuições convergem ~~pra~~ cada a no qual a distribuição de ν_1^0 é contínua, i.e.,

$$\lim_{n \to \infty} P\left[\frac{1}{\sqrt{n}} \sum_{i=1}^n X_i \le a\right] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^a e^{-\frac{t^2}{2}} dt$$

Questão 3

(3) Observe que

$$\int X_j \, d\mathbb{P} = \int_{\{X_j = j^{-1/2}\}} j^{-\frac{1}{2}} d\mathbb{P} + \int_{\{X_j = -j^{-\frac{1}{2}}\}} -j^{-\frac{1}{2}} d\mathbb{P}$$

$$= j^{-\frac{1}{2}} \cdot \frac{1}{2} - j^{-\frac{1}{2}} \cdot \frac{1}{2} = 0,$$ i.e, a média de X_j é $0 = \mu$.

Por outro lado,

$$\int X_j^2 \, d\mathbb{P} = \int_{\{X_j = j^{-1/2}\}} (j^{-1/2})^2 d\mathbb{P} + \int_{\{X_j = -j^{-\frac{1}{2}}\}} (-j^{-\frac{1}{2}})^2 d\mathbb{P}$$

$$= j^{-1} \cdot \frac{1}{2} + j^{-1} \cdot \frac{1}{2} = \frac{1}{j}.$$

Considere $y_j = \sqrt{j} X_j$, então

$$\int y_j^2 d\mathbb{P} = \int j X_j^2 = j \cdot \frac{1}{j} = 1 = \sigma$$

Logo, as variáveis y_j são independentes, tem media $\mu = 0$ e variança $\sigma = 1$.

Pelo Teorema Central do Limite, pode-se afirmar que

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(Questão 3)

... que:

a soma $T_n = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} \overbrace{j^{\alpha} X_j}^{y_j}$ converge fraco* a $\mathcal{N}_0^1$, distribuição normal standar (média 0, variança 1).

(4) Similarmente:

$$\int X_j \, d\mathbb{P} = \int_{\{X_j = j^{\alpha}\}} j^{\alpha} \, d\mathbb{P} + \int_{\{X_j = -j^{\alpha}\}} (-j^{\alpha}) \, d\mathbb{P}$$

$$= j^{\alpha} \cdot \frac{1}{2} - j^{\alpha} \cdot \frac{1}{2} = 0$$

$$\int X_j^2 \, d\mathbb{P} = \int_{\{X_j = j^{\alpha}\}} j^{2\alpha} \, d\mathbb{P} + \int_{\{X_j = -j^{\alpha}\}} j^{2\alpha} \, d\mathbb{P} = j^{2\alpha} \cdot \frac{1}{2} + j^{2\alpha} \cdot \frac{1}{2} = j^{2\alpha}$$

D

$(j^2)^{\alpha}$ $\quad (j^{-\alpha} X_j)^2 = j^{-2\alpha} X_j^2 =$

Questão 1

(1) Seja $A \in M_{n\times n}(\mathbb{R})$, com $\sigma(A) = \{\lambda_1, \lambda_2, \ldots, \lambda_t\}$.
Existe uma base $\mathfrak{J} = J_{b_1} \cup J_{b_2} \cup \ldots \cup J_{b_t}$ para $\mathbb{R}^n$
e uma matriz não-singular $P \in M_{n\times n}(\mathbb{R})$ /

$$P^{-1}AP = J = \text{diag}(J(\lambda_1), \ldots, J(\lambda_t)),$$

onde $J(\lambda_i) = \begin{pmatrix} \lambda_i & 1 & & 0 \\ & \lambda_i & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda_i \end{pmatrix}$.

(2) $\det(tI-A) = t(t^4+2t^2a+a^2)$

$$= t(t^2+a)^2$$

$a=-b^2$, $= t(t-b)^2(t+b)^2$

$\sigma(A) = \{0,-b,b\}$

▷ OPÇÃO 1:

$PAP^{-1} = J$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \begin{matrix} b & 0 \\ & b \end{matrix} & 0 \\ 0 & 0 & \begin{matrix} -b & 0 \\ & -b \end{matrix} \end{pmatrix}$$

▷ OPÇÃO 2:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \begin{matrix} b & 1 \\ & b \end{matrix} & 0 \\ 0 & 0 & \begin{matrix} -b & 0 \\ 0 & -b \end{matrix} \end{pmatrix}$$

▷ OPÇÃO 3.

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \begin{matrix} b & 0 \\ 0 & b \end{matrix} & 0 \\ 0 & 0 & \begin{matrix} -b & 1 \\ 0 & -b \end{matrix} \end{pmatrix}$$

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(3) $P^{-1}BP = B^{-1}$ $BP = PB^{-1}$

$BPB = P$